Transportation Problem

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OBJECTIVES:

To find an optimum transportation schedule in order to

(i) minimize costs/time/expenditure or

(ii) maximize profits /sales /production /revenue of an organization.

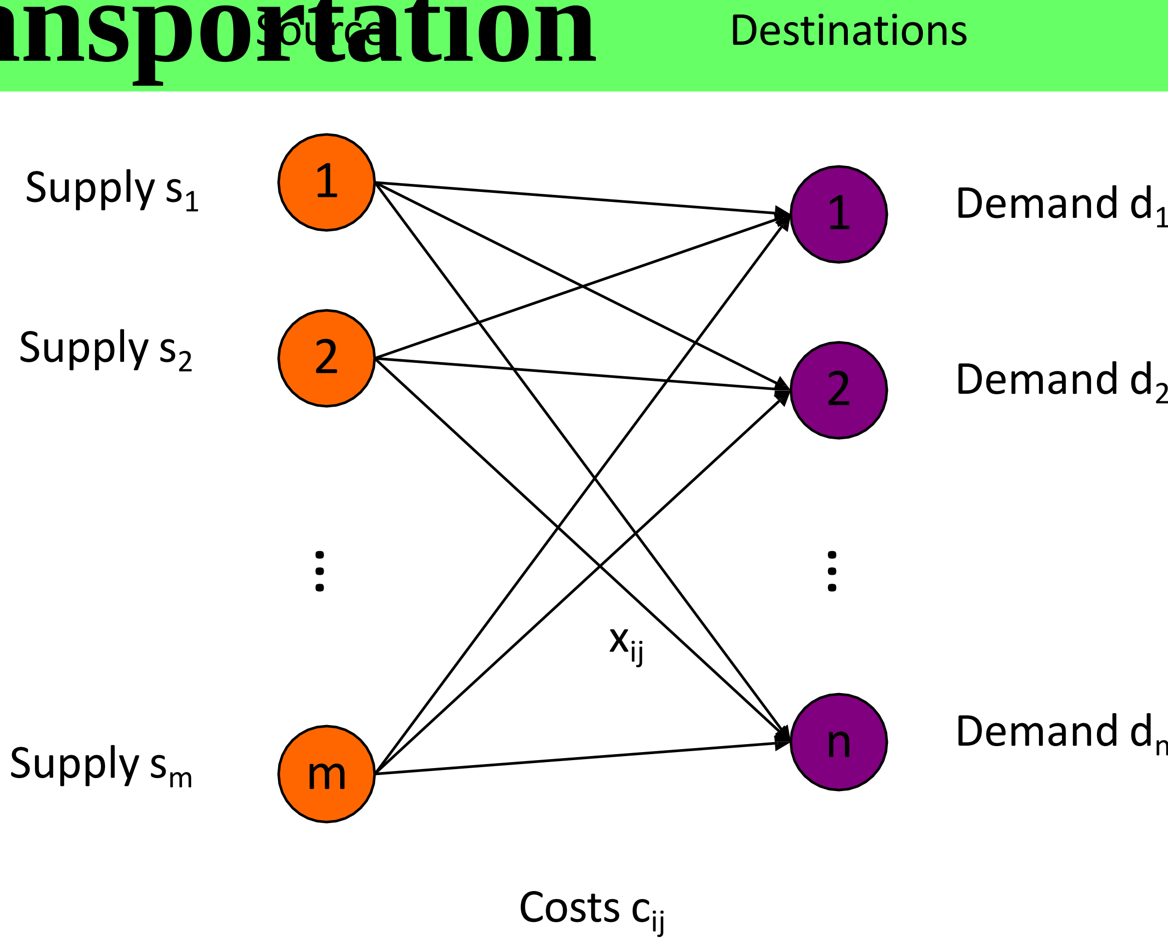
MEANING:

- The transportation problem is a special type of LPP. Here the objective is to minimize the cost or maximize the profits of distributing a product from a number of sources (or origins) to a number of destinations(or markets).

The Transportation Problem

- The problem of finding the minimum-cost distribution of a given commodity
  from a group of supply centers (sources) i=1,…,m to a group of receiving centers (destinations) j=1,…,n
- Each source has a certain supply (si)
- Each destination has a certain demand (dj)

Network Representation of transportation



Basic Terminology:

- Feasible Solution: It is a set of non negative allocations which satisfies the row and column sum condition.
- Basic Feasible Solution: A solution in which number of allocations is equal to one less than the sum of number of rows and columns.
- Optimum Solution: It is a feasible solution which minimizes the cost or maximizes the profits.
- Degenerate Basic Feasible Solution: It is a solution in which number of allocations is less than m+n-1. Where m= number of rows and
- n = number of columns

Assumptions:

- The cost per unit of shipping from a source to a destination is fixed.
- The total cost of shipping from a source to a destination is directly proportional to the number of units shipped
- (= Cost per unit X Units shipped)

Applications of Transportation Problem :

- Minimize shipping costs
- Determine low cost location
- Minimization of Total Cost of Production/Time
- Maximization of profits/production/sales/revenue
- Military distribution system

Types of Transportation Problem:

- **Balanced Transportation Problem** :

(Total Supply of all sources = Total Demand by all destinations)

- **Unbalanced Transportation Problem :**

(Total Supply of all sources ≠ Total Demand of by destinations)

- **Prohibitive Routes Transportation Problem :**

(When Supplies from any source to any destination is prohibited or not possible due to any reason).

- **Multiple solution Problem:**

A problem with more than one optimal solution is called Multiple solution Problem

Phases of Solution to Transportation problem:

- Phase I- Obtain initial basic feasible solution .
- Phase II- Remove degeneracy if any.
- Phase III-Obtain the optimal solution.

Phase I

Methods to find Initial Basic Feasible Solutio

- North West Corner Rule (NWCR)
- Row Minima Method
- Column Minima Method
- Least Cost Method or Least Entry Method
- Vogel's Approximation Method (VAM) or Penalty Method

Out of all these methods Vogel's Approximation Method is the best method.

Vogel's Approximation Method

1. Find the penalties for each row and column by taking the difference of two minimum respective row or column.

2. Identify the row/column with largest penalty . If there are more than one row or column have the same largest penalty, choose any of those arbitrarily.

3. Identify a cell with minimum cost in the row/ column identified in above step. If there are more than one cell with minimum cost in the identified row or column, choose any of those arbitrarily .

4. Allocate maximum number of units in the identified cell keeping in view the row and column constraints. Remove the row/column from further consideration whose supply/demand is exhausted.

5. Again compute the penalties with the reduced transportation table.

PHASE II

Checking for Degeneracy

1. No. Degeneracy if:

Number of allocations= m+n-1

2. **Degeneracy if:**

m+n-1 > Number of allocations

Before proceeding to phase III degeneracy is removed by introducing as many (m+n-1) - number of allocations, infinitely small quantity/s (represented by e or θ) in unoccupied cell/s

(i) With minimum cost/s and

(ii) In independent position/s (i.e. no closed loop is formed from it/them)

PHASE III

Methods to find Optimum Solution

1. Stepping Stone Method

2. Modified Distribution Method or MODI method

Stepping Stone Method:

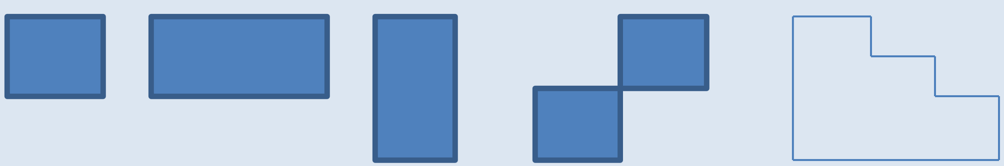
1. Select any unoccupied cell to calculate its net evaluation or opportunity cost.
2. Beginning at this cell, form a closed loop back to the original cell via occupied cells. For making a closed loop

(i) we can move to RHS, LHS, Vertically up and vertically down

(ii) We can cross any number of occupied or unoccupied cells, if need be

(iii) Bend is always at 90° and at filled cell.

Generally loops are of the shapes :



Stepping Stone Method Contd...

3. Put plus sign in the starting unoccupied cell and put minus and plus alternate corners. (There can never be two consecutive plus or minus signs)

4. Calculate an improvement index or net evaluation by adding the unit-cost figures in the cells lying at the plus signs and subtracting the unit costs in the cells lying at minus signs

5.Calculate improvement indices/ net evaluations for all unoccupied cells by using steps from 1 to 4.

6. There will be three possibilities:

A) All improvement indices are > 0, there is optimal solution and it is unique.

B) All improvement indices are ≥ 0, there is optimal solution but alternate solutions to the given problem exist

Stepping Stone Method Contd…

c) If some improvement index/indices is/are negative, there is no optimal solution to the problem at this stage and the solution can be improved. For improvement:

(i) Identify the unoccupied cell with most negative net evaluation.

(ii) Form a closed loop from it. Put plus minus signs at alternate corners starting with plus sign in the starting cell.

(iii) Find the allocations in the negative corner cells.

(iv) Choose minimum allocation out of these negative corner cells. Add it in positive corner cell allocations and subtract it from negative corner cell allocations . Get a new matrix and again check for optimality by using steps 1 to 6. This process will continue till we get all net evaluations ≥ 0.

An Illustration

FROM	TO	Chandigarh	Amritsar	Agra	CAPACITY
New Delhi		15	14	13	100
Calcutta		18	14	13	300
Mangalore		19	17	15	300
DEMAND		300	200	200	700

Initial Feasible Solution using Northwest Corner Rule

FROM \ TO	A. Chandigarh	B. Amritsar	C. Agra	FACTORY CAPACITY
D. New Delhi	15 100	14	13	100
E. Calcutta	18 200	14 100	13	300
F. Mangalore	19	17 100	15 200	300
WAREHOUSE DEMAND	300	200	200	700

IFS= DA + EA +EB + FB + FC = 100(15) + 200(18) + 100(14) + 100(17) + 200(15)
= 1500 + 3600 + 1400 + 1700 + 3000 = 11200

Optimizing Solution using Stepping-Stone Method

From \ To	(A) Chandigarh	(B) Amritsar	(C) Agra	Factory capacity
(D) New Delhi	15 100 -	14 +	13	100
(E) Calcutta	18 200 +	14 - 100	13	300
(F) Mangalore	19	17	15	300
Warehouse requirement	300	200	200	700

New Delhi-Amritsar Net Evaluation

= 14 - 14+ 18 - 15

= +3

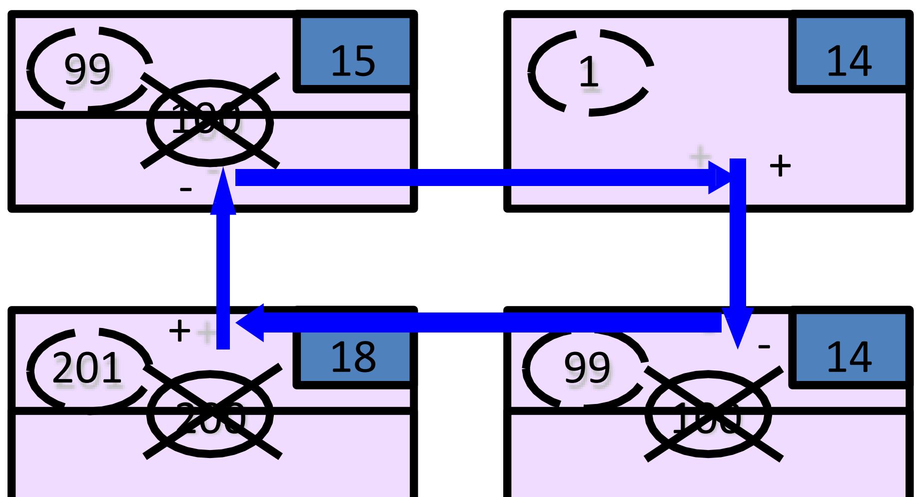


Figure C.5

Net Evaluation for Cell CD

FROM \ TO	A. Chandigarh	B. Amritsar	C. Agra
D. New Delhi	15 100 -	14	13 +
E. Calcutta	18 200 +	14 100 -	13
F. Mangalore	19	17 100 +	15 200 -

New Delhi- Agra Net Evaluation

= 13 - 15 + 17 - 14 + 18 - 15 = + 4

Net Evaluation for Cells EC & FA

From \ To	(A) Chandigarh	(B) Amritsar	(C) Agra	Factory capacity
(D) New Delhi	100 (15)	(14)	(13)	100
(E) Calcutta	200 (18)	100 (14)	(13)	300
(F) Mangalore	(19)	100 (17)	200 (15)	300
Warehouse requirement	300	200	200	700

Calcutta- Agra Net Evaluation

Cell EC = 13 - 15 + 17 - 14 = +1 (Closed path = EC - FC + FB - EB)

Mangalore- Chandigarh Net Evaluation

Cell FA = 19 – 18 + 14 - 17 = -2 (Closed path = FA - EA + EB - FB)

Stepping Stone Method Contd..

Since one net evaluation is negative , the current solution is not optimal. For improvement:

1. Form a closed loop FA EA EB FB from cell FA having most negative net evaluation (= -2).
2. Allocations at negative corners are 200(EA) and 100(FB). Minimum out of these is 100 so 100 will be added in 0(Allocation in positive corner cell FA) and 100 (Allocation in positive corner cell EB)
3. After new allocations the new matrix will like as in next slide and the new total cost will be 11000 (11200 -2x100 = old total cost –negative net evaluation x number of units reallocated)

Stepping-Stone Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	100 [5]	[4]	[3]	100
(E) Evansville	200 [8] -	100 [4] +	[3]	300
(F) Fort Lauderdale	+ [9]	100 [7] -	200 [5]	300
Warehouse requirement				

1. Add 100 units on route FA
2. Subtract 100 from routes FB
3. Add 100 to route EB
4. Subtract 100 from route EA

Figure C.7

Stepping-Stone Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	100 [15]	[14]	[13]	100
(E) Evansville	100 [18]	200 [14]	[13]	300
(F) Fort Lauderdale	100 [19]	[17]	200 [15]	300
Warehouse requirement	300	200	200	700

Total Cost = 15(100) + 18(100) + 14(200) + 19(100) + 15(200)
= 11,000

Figure C.8

Contd…..

Check for optimality .

Net Evaluation for cell

BD: 14-14+18-15=3 (Loop: BD,BE,AE,AD)

CD: 13-15+19-15 =2 (Loop: CD,CF,AF,AD)

CE: 13-15+19-18= -1 (Loop: CE,CF,AF,AE)

BF: 17-19+18-14= 2 (Loop: BF,AF,AE,BE)

Since Net evaluation of cell CE is negative so still it is not an optimum solution.

Forming a closed loop from CE (CE,CF,AF,AD)

Minimum allocation of negative corner allocations is 100. Thus new redistribution will give us a new matrix as

Stepping-Stone Method

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	100 [15]	[14]	[13]	100
(E) Evansville	[18]	200 [14]	100 [13]	300
(F) Fort Lauderdale	200 [19]	[17]	100 [15]	300
Warehouse requirement	300	200	200	700

Total Cost $= 15(100) + 14(200) + 13(100) + 19(200) + 15(100)$

$= 10{,}900$

Checking for optimality:
Net Evaluation for cell
BD: +14-14+13-15+19-15=2 (Loop: BD,BE,CE,CF,AF,AD)
CD: + 13- 15+19- 15 = 2 (Loop: CD,CF,AF,AD)
AE: 18-13+ 15-19 =1 (Loop: AE,CE,CF,AF)
BF: 17 -14 +13-15 +1 (Loop: BF,BE,CE,CF)
Since all net evaluations are positive. Hence it is an optimal solution.
Since all net evaluations are positive and none of them is zero. Hence this optimal solution is unique.
Total Cost =15(100) + 14(200) + 13(100) + 19(200) + 15(100)
= 10,900

Modified Distribution Method

1. Assign (m + n) dummy variables . Assign one dummy variable per r
(U_1 U_2 U_3 ……. etc.)and one dummy variable per column(V_1, V_2 , V_3 etc).
2. Frame equations involving dummy variables. One equation for each occupied cell such that $Ui + V_j = C_{IJ}$
3. Take any U or V equal to zero. And find the unknown values of left out U's and V's.
4. Calculate net evaluations for each unoccupied cell by using
 Net Evaluation of unoccupied cell in ith row and jth column
$= C_{IJ} - (Ui + V_j)$

5. There will be three possibilities:

 A) All net evaluations are > 0, there is optimal solution and it is unique.

B) All net evaluations are ≥ 0, there is optimal solution but alternate solutions to the given problem exist

Conti…….

C) If some net evaluations / improvement index/indices are negative, there is no optimal solution to the problem at this stage and the solution can be improved. For improvement:

(i) Identify the unoccupied cell with most negative net evaluation.
(ii) Form a closed loop from it. Put plus minus signs at alternate corners starting with plus sign in the starting cell.
(iii) Find the allocations in the negative corner cells.
(iv) Choose minimum allocation out of these negative corner cells. Add it in positive corner cell allocations and subtract it from negative corner cell allocations . Get a new matrix and again check for optimality by using steps 1 to 6. This process will continue till we get all net evaluations either >0 or ≥ 0.

Special Issues in Modeling

- ☑ Demand not equal to supply
  - ☑ Called an unbalanced problem
  - ☑ Common situation in the real world
  - ☑ Resolved by introducing dummy sources or dummy destinations as necessary with cost coefficients of zero

From \ To	V_1	V_2	V_3	Factory capacity
U_1	100 (15)	(14)	(13)	100
U_2	200 (18)	100 (14)	(13)	300
	(19)	100 (17)	200 (15)	300
Warehouse requirement	300	200	200	700

Calculation of values of dummy variables	Net Evaluation=C_{IJ} - (Ui + $V_{j,}$)
Let $U_1 = 0$ $C_{11} = U_1 + V_1$ $15 = 0 + V_1 \quad \therefore V_1 = 15$ $C_{21} = U_2 + V_1$ $18 = U_2 + 15 \quad \therefore U_2 = 3$ $C_{22} = U_2 + V_2$ $14 = 3 + V_2 \quad \therefore V_2 = 11$ $C_{32} = U_3 + V_2$ $17 = U_3 + 11 \quad \therefore U_3 = 6$ $C_{33} = U_3 + V_3$ $15 = 6 + V_3 \quad \therefore V_3 = 9$	For cell12 $=C_{12}$ - (U1 + V2) = 14-(0+11) =3 For cell13 $=C_{13}$ - (U1 + V3) = 13-(0+9) =4 For cell23 $=C_{23}$ - (U2 + V3) = 13-(3+9) =1 For cell31 $=C_{31}$ - (U3 + V1) = 19-(6+15) = -2 (Note: These are same as under stepping Stone Method)

Improvement of the current solution

Since one net evaluation, for cell 31, is negative , the current solution is not optimal. For improvement:

1. Form a closed loop starting from cell 31 (from cells 31,21, 22 and 32) .

2. Allocations at negative corners are 200(cell 21) and 100(cell 32). Minimum out of these is 100, so 100 will be added in 0 (Allocation in positive corner cell 31) and in 100 (Allocation in positive corner cell 22)

3. After new allocations the new matrix will like as in next slide and the new total cost will be 11000 (11200 -2x100 = old total cost –negative net evaluation x number of units reallocated)

MODIFIED DISTRIBUTION METHOD

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
U	100 [15]	[14]	[13]	100
(E) Evansville	100 [18]	200 [14]	[13]	300
(F) Fort Lauderdale	100 [19]	[17]	200 [15]	300
Warehouse requirement	300	200	200	700

Total Cost = 15(100) + 18(100) + 14(200) + 19(100) + 15(200)
= 11,000

Calculation of values of dummy variables	Net Evaluation= C_{IJ} - (U_i + $V_{j,}$)
Let $U_1 = 0$ $C_{11} = U_1 + V_1$ $15 = 0 + V_1$ ∴ $V_1 = 15$ $C_{21} = U_2 + V_1$ $18 = U_2 + 15$ ∴ $U_2 = 3$ $C_{22} = U_2 + V_2$ $14 = 3 + V_2$ ∴ $V_2 = 11$ $C_{32} = U_3 + V_2$ $17 = U_3 + 11$ ∴ $U_3 = 6$ $C_{33} = U_3 + V_3$ $15 = 6 + V_3$ ∴ $V_3 = 9$	For cell12 $= C_{12}$ - (U1 + V2) = 14-(0+11) =3 For cell13 $= C_{13}$ - (U1 + V3) = 13-(0+9) =4 For cell23 $= C_{23}$ - (U2 + V3) = 13-(3+9) =1 For cell31 $= C_{31}$ - (U3 + V1) = 19-(6+15) = -2 (Note: These are same as under stepping Stone Method)

Thank you